One Trade to Move Them All: Optimal Price Impact for Minimal-Cost Execution

Andrea Bortolini

Draft note

1 Motivation

Standard optimal execution treats price impact as a cost. A trader who needs to buy or sell a large quantity over a short horizon usually faces a tradeoff: execute quickly and pay more impact, or split the order and reduce execution cost at the expense of timing risk.

This note studies a simple extension: what if the trader can simultaneously trade a related digital claim whose payoff depends on the same price that the trader's order affects?

For example, a trader who must buy a large quantity of BTC may push the short-horizon reference price upward. If a digital market pays when BTC finishes above a reference level, the trader's own execution can increase the probability that the digital pays. In that case, execution impact is not only a cost; it also changes the payoff distribution of a contingent claim.

The question is:

When does the ability to trade a same-direction digital claim change the optimal execution policy?

The goal is not to claim that impact is eliminated. The trader still pays execution costs. Rather, the model asks whether a segmented or slow-moving digital market can transfer part of that cost back to the trader through an impact-sensitive payoff.

2 Basic Setup

A trader has an exogenous need to trade quantity Q of an underlying asset over a short horizon. Let d denote trade direction:

$$d = +1 \quad \text{for a buy}, \qquad d = -1 \quad \text{for a sell}.$$

The trader has two possible execution policies.

Strategy A: standard split execution. The trader splits the order across two periods to reduce price impact.

Strategy B: concentrated execution with digital overlay. The trader executes the full order immediately and simultaneously buys a same-direction digital:

- if buying the underlying, buy an Up digital;
- if selling the underlying, buy a Down digital.

The simultaneous-execution assumption is the clean base case. It means the digital order is entered before the digital market fully observes or incorporates the trader's underlying execution. Later extensions can relax this with latency, leakage, partial observability, and digital-market repricing.

3 Price Impact and Settlement

Let p_0 be the pre-trade reference price. Without the trader's order, the short-horizon settlement price would be:

$$p_1 = p_0 + \mu + \sigma\varepsilon.$$

With the trader's concentrated order, the settlement price becomes:

$$p_1^Q = p_0 + \mu + d\beta Q + \sigma\varepsilon.$$

Here μ is the normal short-horizon drift, $\sigma\varepsilon$ is unrelated price noise, and βQ is the settlement-relevant price impact generated by the trader's order. The term $d\beta Q$ moves the price in the direction of the trade.

The digital pays if the settlement price moves in the trader's direction relative to a threshold K:

$$D = \mathbf{1}_{\{d(p_1^Q - K) > 0\}}.$$

For a buy order, this is an Up digital. For a sell order, this is a Down digital. The key point is simple: the trader's execution shifts the settlement distribution of the digital payoff.

4 Digital Mispricing Wedge

If the digital market prices the claim using the pre-trade distribution, its price reflects the payout probability without the trader's impact. Call this market-implied probability π_0.

Conditional on the trader executing Q, the true payout probability is higher:

$$\pi_Q = \mathbb{P}\Big(d(p_1^Q - K) > 0\Big).$$

The trader's edge per digital contract is:

$$e(Q) = \pi_Q - \pi_0.$$

This wedge exists only if the digital market is segmented or otherwise does not fully condition on the trader's simultaneous underlying execution. If the digital market perfectly anticipates the trader's flow, then π_0 adjusts to π_Q, and the edge disappears.

5 Execution Tradeoff

Concentrated execution is more expensive than split execution because it creates more price impact. Let $\Delta(Q)$ denote the extra execution cost of trading all at once instead of using the optimal split schedule.

The digital overlay is useful only if the expected gain from the digital exceeds this extra execution cost:

$$Ne(Q) \geq \Delta(Q),$$

where N is the number of digital contracts purchased.

This is the central condition of the model. Concentrated execution becomes privately optimal when the gain from shifting the digital payoff probability is larger than the extra impact cost from concentrating the underlying trade.

This does not mean the trader avoids price impact. It means the trader may be compensated for part of it through a related contingent-claim market.

6 Digital Market Depth

Without frictions, if $e(Q) > 0$, a risk-neutral trader would want an arbitrarily large digital position. That is unrealistic. The model therefore needs digital-market depth.

Assume the trader's own digital demand moves the digital price:

$$\pi(N) = \pi_0 + \eta_B N.$$

Here η_B measures digital-market illiquidity. A high η_B means the digital price moves quickly against the trader, so available depth is poor. A low η_B means the digital market is deep.

The expected gain from the digital overlay is then approximately:

$$G(N; Q) = Ne(Q) - \frac{1}{2}\eta_B N^2.$$

This creates a finite optimal digital position. The strategy is strongest when:

- the underlying order has large settlement-relevant impact;
- the digital market is slow or segmented;
- the digital book is deep enough to absorb meaningful size;
- the settlement horizon is short enough that impact affects the reference price;
- the extra cost of concentrating the underlying trade is not too large.

7 Interpretation and Extensions

The model changes the usual execution problem. In standard execution, price impact is purely a cost. Here, price impact also changes the payoff probability of a related digital claim. This creates a new tradeoff: execution cost vs digital payoff probability shift.

The base model assumes simultaneous execution and no information leakage. Natural extensions include latency and sequencing, partial observability of underlying flow, endogenous digital repricing, transient versus permanent impact, reference-price design, and regulatory or market-design constraints.

The same mechanism can also be interpreted as a market-design problem. A digital market is fragile when its settlement variable can be moved by trades in the underlying market, but the digital price does not fully internalize that possibility.

8 Main Takeaway

If a trader's required execution affects the settlement price of a related digital claim, then optimal execution may involve a simultaneous same-direction digital overlay.

The strategy requires segmented pricing, finite but meaningful digital depth, and enough underlying impact to shift the settlement probability. But when those conditions hold, the optimal policy may be more concentrated than standard execution theory would suggest.

In that sense, price impact is no longer only a trading cost. It becomes an input into the payoff distribution of another market.